Equivalence between Born-Infeld tachyon and effective real scalar field theories for brane structures in warped geometry

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Abstract

An equivalence between Born-Infeld and effective real scalar field theories for brane structures is built in a warped space-time scenario. Once the equations of motion for tachyon fields related to the Born-Infeld action can be written as first-order equations, a simple analytical connection with real scalar field superpotentials can be found. This equivalence leads to the conclusion that both systems can support identical thick brane solutions as well as brane structures described through localized energy densities, $T_{00}(y)$, in the 5th dimension, y. Our results indicate that thick brane solutions realized by the Born-Infeld cosmology can be connected to real scalar field brane scenarios which can be used to effectively map the tachyon condensation mechanism.

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Brane cosmology driven by scalar fields have been recurrently studied in order to approach the cosmological constant and hierarchy problems [1–3] as well as symmetry breaking issues [4] (see also Ref. [5] for the projection on the brane of vector and tensor fields in the bulk space). The first ideas for brane world scenarios assumed a warped 4-dimensional brane universe embedded in a higher dimensional bulk, where the brane corresponds to a localized delta function of the extra dimension coordinate [3].

The brane scenario examined here is related to generic solutions of the 5-dimensional Born-Infeld field theories of the form

$$S = \int dx^5 \sqrt{\det g_{AB}} \left[-\frac{1}{4} R - U(\varphi) \sqrt{1 - g^{AB} \partial_A \varphi \partial_B \varphi} \right], \tag{1}$$

where R is the scalar curvature, and g_{AB} denotes the metric tensor, with A and B running from 0 to 4. The field φ is a tachyon field and $U(\varphi)$ is its potential, with dimensional constants absorbed by the field normalization. From this action, it has been conjectured that the dynamics of a Born-Infeld tachyon field in a background of an unstable D-brane system can be perturbatively described by the dynamics of an effective real scalar field [6]. According to such an assumption, tachyon calculations would be reliable only in the approximation where φ derivatives can be truncated beyond the quadratic order [7].

The perturbative truncation leads to an effective action driven by a real scalar field, χ, coupled to 5-dimensional gravity, given by

$$S^{\text{eff}} = \int dx^5 \sqrt{\det g_{AB}} \left[-\frac{1}{4} R + \frac{1}{2} g_{AB} \partial^A \chi \partial^B \chi - V(\chi) \right], \tag{2}$$

which gives rise to a series of possibilities for investigating the related tachyon field dynamics. In quantum field theories, a tachyon field can be realized by the instability of the quantum vacuum, described by the quantum state displaced from a local maximum of an effective potential like $V(\chi)$. In the effective real scalar field scenario, the tachyon field would follow a spontaneous symmetry breaking (SSB) that implies into a process dubbed as tachyon condensation [8, 9]. Considering its remarkable applications in brane world models, tachyon condensation can play an important role also in string theory (see e. g. Refs. [10, 11]). Tachyon condensation can also reproduce the results of a collision process similar to a kink-antikink or to a soliton-antisoliton annihilation that drives the system to the SSB vacuum after complete annihilation. In this context, the Big-Bang has been hypothesized to occur as a result of such a brane-antibrane collision. Notice that branes defined as classical solutions

of the tachyonic potentials naturally arise in systems with rolling tachyons on unstable branes [12]. The resulting vacuum state after annihilation exhibits the remaining lower-dimensional branes as relics of tachyon condensation [13] that (re)produce the effects of cosmic strings in brane cosmology [14–16].

Real scalar field models coupled to gravity lead also to analytical solutions of gravitating defect structures which allow for the inclusion of thick branes used in several brane cosmology scenarios. Thick domain walls, for instance, are often associated to solvable integrable models. In general, these potentials associated to single real scalar field support BPS type solutions [25, 26] of first-order differential equations. In this case, the equations result into topological defects that admit internal structures.

However, there has been no consensus about how reliably effective real scalar field models describe the Born-Infeld tachyonic dynamics [17], despite of the importance of real scalar fields [18, 19] in describing brane structures in warped geometry [20–24].

Therefore, the brane model discussed in this letter treats Born-Infeld tachyon fields without any build in association with the real scalar field (c. f. Eq. (2)). Assuming that the equations of motion for Born-Infeld tachyon fields can be mapped by superpotential parameters constrained by first-order equations, analogously to the procedure of mapping BPS solutions into real scalar fields, one is able to find exact solutions for the tachyon field, φ. In addition, a fruitful connection between tachyon and real scalar field superpotentials can be identified. The resulting brane scenario exhibits an exact equivalence between Born-Infeld tachyon and real scalar field dynamics in 5-dimensions, which is reproduced by a unique warp-factor and leads to the same localized energy densities.

In what follows we shall call χ a real scalar field, even when considering that its associated action may approach a tachyonic action that circumstantially results into a condensation mechanism and associated instabilities. We shall bear in mind that we seek for an analytical correspondence of the Born-Infeld tachyon with the real scalar field in order to obtain two equivalent brane world scenarios.

The framework for discussing a single real scalar field coupled to gravity in the brane scenario follows previous discussions [20–24]. The correspondence between the Born-Infeld tachyon and the real scalar field is obtained through a set of first-order equations. Novel analytically integrable models that admit thick brane solutions to the Born-Infeld action through twin warp factors bound from above are also discussed.

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Real scalar fields

Let us start describing the cosmological setting of our proposal. We consider a 5-dimensional space-time warped in 4-dimensions. In order to ensure the Poincaré invariance in 4-dimensions, the space-time metric is written as follows,

$$ds^2 = g_{AB}\, dx^A\, dx^B = e^{2A(y)}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu - dy^2, \tag{3}$$

where $\eta_{\mu\nu} \equiv \{+1, -1, -1, -1\}$, μ and ν run from 0 to 3, $y \equiv x_4$ is the $5th$ dimension coordinate that corresponds to the brane position, $e^{2A(y)}$ is the warp factor, and the brane tension is set equal to unit, $\sigma = 1$.

Considering the real scalar field action, Eq. (2), one can compute the stress-energy tensor

$$T^{\chi}_{AB} = \partial_A \chi \partial_B \chi + g^{AB}\, V(\chi) - \frac{1}{2} g_{AB}\, g^{MN} \partial_M \chi \partial_N \chi, \tag{4}$$

which, supposing that both the scalar field and the warp factor dynamics depend only on the extra coordinate, y, leads to an explicit dependence of the energy density in terms of the field, χ, and of its first derivative, $d\chi/dy$, as

$$T^{\chi}_{00}(y) = \left[\frac{1}{2} \left(\frac{d\chi}{dy} \right)^2 + V(\chi) \right] e^{2A(y)}. \tag{5}$$

With the same constraints on the χ dependence on y, the equations of motion arising from the above action are

$$\frac{d^2\chi}{dy^2} + 4 \frac{dA}{dy} \frac{d\chi}{dy} - \frac{d}{d\chi} V(\chi) = 0, \tag{6}$$

$$\frac{3}{2} \frac{d^2 A}{dy^2} = - \left(\frac{d\chi}{dy} \right)^2, \tag{7}$$

$$3 \left(\frac{dA}{dy} \right)^2 = \frac{1}{2} \left(\frac{d\chi}{dy} \right)^2 - V(\chi). \tag{8}$$

If the potential for the real scalar field can be written in terms of a *superpotential*, w, then one has

$$V(\chi) = \frac{1}{8} \left(\frac{dw}{d\chi} \right)^2 - \frac{1}{3} w^2, \tag{9}$$

that leads to the first-order equations

$$\frac{d\chi}{dy} = \frac{1}{2} \frac{dw}{d\chi}, \tag{10}$$

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and

$$\frac{dA}{dy} = -\frac{1}{3}w, \tag{11}$$

which yield a solution that was first discussed when studying supergravity on (non)supersymmetric domain walls [21, 27, 28], from which follows an energy density expressed as

$$T_{00}^{\chi}(y) = \left[\frac{1}{4}\left(\frac{dw}{d\chi}\right)^2 - \frac{1}{3}w^2\right] e^{2A(y)}. \tag{12}$$

As will be discussed next, an analogous first-order formulation for tachyon fields can be carried out.

Born-Infeld tachyon fields

The action for a tachyon field, φ, coupled to 5-dimensional gravity is given by Eq. (1), in the geometry described by Eq. (3). The tachyon field and the warp factor depend only on y and allow for computing the stress-energy tensor

$$T_{AB}^{\varphi}(y) = U(\varphi)\,\partial_A\varphi\partial_B\varphi\frac{1}{\sqrt{1 - g^{MN}\partial_M\varphi\partial_N\varphi}} + g_{AB}\,U(\varphi)\,\sqrt{1 - g^{MN}\partial_M\varphi\partial_N\varphi}, \tag{13}$$

from which one also obtains the energy density as

$$T_{00}^{\varphi}(y) = e^{2A(y)}\,U(\varphi)\,\sqrt{1 + \left(\frac{d\varphi}{dy}\right)^2}. \tag{14}$$

Under the same assumptions about the φ dependence on y, the equations of motion can be written as

$$\frac{d^2\varphi}{dy^2} + \left[1 + \left(\frac{d\varphi}{dy}\right)^2\right]\left(4\frac{dA}{dy}\frac{d\varphi}{dy} - \frac{1}{U(\varphi)}\frac{d}{d\varphi}U(\varphi)\right) = 0, \tag{15}$$

$$\frac{3}{2}\frac{d^2A}{dy^2} = \left(\frac{d\varphi}{dy}\right)^2\frac{U(\varphi)}{\sqrt{1 + \left(\frac{d\varphi}{dy}\right)^2}}, \tag{16}$$

$$3\left(\frac{dA}{dy}\right)^2 = -\frac{U(\varphi)}{\sqrt{1 + \left(\frac{d\varphi}{dy}\right)^2}}. \tag{17}$$

Thus, once a potential for the tachyon field can be written as, for instance,

$$U(\varphi) = -\frac{3}{v^2}\sqrt{1 + \frac{1}{4}\left(\frac{dv}{d\varphi}\right)^2}, \tag{18}$$

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where another *superpotential*, v, is introduced, one obtains the first-order equations,

$$\frac{d\varphi}{dy} = \frac{1}{2}\frac{dv}{d\varphi},$$

(19)

and

$$\frac{dA}{dy} = -\frac{1}{v},$$

(20)

such that the energy density Eq. (14) can be written as

$$T_{00}^{\varphi}(y) = -\frac{3}{v^2}\left[1 + \frac{1}{4}\left(\frac{dv}{d\varphi}\right)^2\right]e^{2A(y)}.$$

(21)

The energy densities Eq. (5) and Eq. (14) can be shown to be the same through the relationship between the superpotentials, w and v,

$$v(y)w(y) = 3.$$

(22)

This relationship results into an equivalence between the Born-Infeld tachyon and the real scalar field dynamics. Indeed, from Eqs. (11) and (20), one obtains

$$\left(\frac{d\chi}{dy}\right)^2 = -3\left(\frac{dA}{dy}\right)^2\left(\frac{d\varphi}{dy}\right)^2,$$

(23)

through which, from Eqs. (12) and (21), and after some straightforward mathematical manipulations, it follows that

$$
\begin{aligned}
T_{00}^{\varphi}(y) &= -\frac{3}{v^2}\left[1 + \frac{1}{4}\left(\frac{dv}{d\varphi}\right)^2\right]e^{2A(y)} \\
&= -3\left(\frac{dA}{dy}\right)^2\left[1 + \left(\frac{d\varphi}{dy}\right)^2\right]e^{2A(y)} \\
&= \left[\left(\frac{d\chi}{dy}\right)^2 - 3\left(\frac{dA}{dy}\right)^2\right]e^{2A(y)} \\
&= \left[\frac{1}{4}\left(\frac{dw}{d\chi}\right)^2 - \frac{1}{3}w^2\right]e^{2A(y)} \\
&= T_{00}^{\chi}(y).
\end{aligned}
$$

(24)

To illustrate such an equivalence between two distinct models for brane structures, let us consider two examples, I and II, for which the warp factor, $A(y)$, and the energy density, $T_{00}(y)$, can be analytically computed.

In terms of a real scalar, a model I is introduced through a sine-Gordon inspired super-potential given by

$$w^I(\chi) = \frac{2}{\sqrt{2}a} \sin\left(\sqrt{\frac{2}{3}}\chi\right), \tag{25}$$

which reproduces the results previously obtained in Ref. [22]. The model II consists in a deformed $\lambda\chi^4$ theory with the superpotential

$$w^{II}(\chi) = \frac{3\sqrt{3}}{a}\left(1 - \frac{\chi^2}{9}\right). \tag{26}$$

In both cases, a is an arbitrary parameter to fix the thickness of the brane described by the warp factor, $e^{2A(y)}$. As expected, through Eq. (10), the superpotentials w^I and w^{II} lead to the respective solutions for $\chi(y)$,

$$\chi^I(y) = \sqrt{6}\arctan\left[\tanh\left(\frac{y}{2\sqrt{2}a}\right)\right], \tag{27}$$

and

$$\chi^{II}(y) = 3\operatorname{sech}\left(\frac{\sqrt{3}y}{2a}\right), \tag{28}$$

where, for convenience, we have just considered the positive solutions.

The corresponding expressions for the warp factor are obtained from Eq. (10) and are respectively,

$$A^I(y) = -\ln\left[\cosh\left(\frac{y}{\sqrt{2}a}\right)\right], \tag{29}$$

$$A^{II}(y) = \tanh\left(\frac{\sqrt{3}y}{2a}\right)^2 - 2\ln\left[\cosh\left(\frac{\sqrt{3}y}{2a}\right)\right], \tag{30}$$

where we have adopted the normalization criterium that sets $A(0) = 0$. The solutions for A^I and A^{II} are depicted in Fig. 1. One can observe that both models I and II give rise to thick branes with the corresponding localized energy densities (c. f. Eq. (12)) given respectively by

$$T_{00}^I(y) = \frac{3}{4a^2}\operatorname{sech}\left(\frac{y}{\sqrt{2}a}\right)^2\left[\operatorname{sech}\left(\frac{y}{\sqrt{2}a}\right)^2 - 2\tanh\left(\frac{y}{\sqrt{2}a}\right)^2\right], \tag{31}$$

and

$$T_{00}^{II}(y) = \frac{9}{a^2}e^{2\tanh\left(\frac{\sqrt{3}y}{2a}\right)^2}\left[\operatorname{sech}\left(\frac{\sqrt{3}y}{2a}\right)\tanh\left(\frac{\sqrt{3}y}{2a}\right)\right]^2\left[\frac{3}{4}\operatorname{sech}\left(\frac{\sqrt{3}y}{2a}\right)^4 - \tanh\left(\frac{\sqrt{3}y}{2a}\right)^4\right], \tag{32}$$
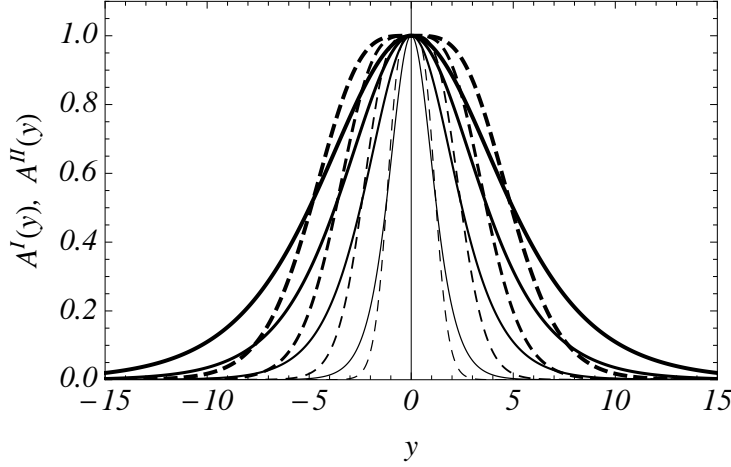
FIG. 1: Warp factor, $e^{2A(y)}$, for models I (solid lines) and II (dashed lines) for a parameter a running from 1 (thinest line) to 4 (thickest line), implying an increasing thickness.

which are depicted in Fig. 2.

The two Born-Infeld models are obtained via the corresponding superpotentials, $v^{I,II}(y)$, through the constraint Eq. (22). They satisfy the following first-order equations for φ,

$$\frac{d\varphi^I}{dy} = \pm \frac{i}{\sqrt{2}} \frac{1}{\sinh{(y/\sqrt{2}a)}}, \tag{33}$$

and

$$\frac{d\varphi^{II}}{dy} = \pm \frac{\sqrt{3}i}{2} \frac{\cosh{(\sqrt{3}y/2a)}}{\left[\sinh{(\sqrt{3}y/2a)}\right]^2}, \tag{34}$$

whose corresponding solutions are respectively:

$$\varphi^I(y) = \pm i\, a \, \ln\left[\tanh\left(\frac{y}{2\sqrt{2}a}\right)\right], \tag{35}$$

$$\varphi^{II}(y) = \mp i\, a \, \mathrm{csch}\left(\frac{\sqrt{3}y}{2a}\right). \tag{36}$$

Finally, the corresponding Born-Infeld tachyon potentials are given respectively by

$$U^I(\varphi) = -\frac{3}{2\sqrt{2}a^2} \sec\left(\frac{\varphi}{a}\right) \left[2\sec\left(\frac{\varphi}{a}\right)^2 + \tan\left(\frac{\varphi}{a}\right)^2\right]^{\frac{1}{2}}, \tag{37}$$

and

$$U^{II}(\varphi) = -\frac{9}{a^2} \left[1 + \frac{3}{2}\frac{\varphi^2}{a^2}\left(1 - \frac{\varphi^2}{a^2}\right)\right]^{\frac{1}{2}} \left(1 - \frac{\varphi^2}{a^2}\right)^{-3}, \tag{38}$$
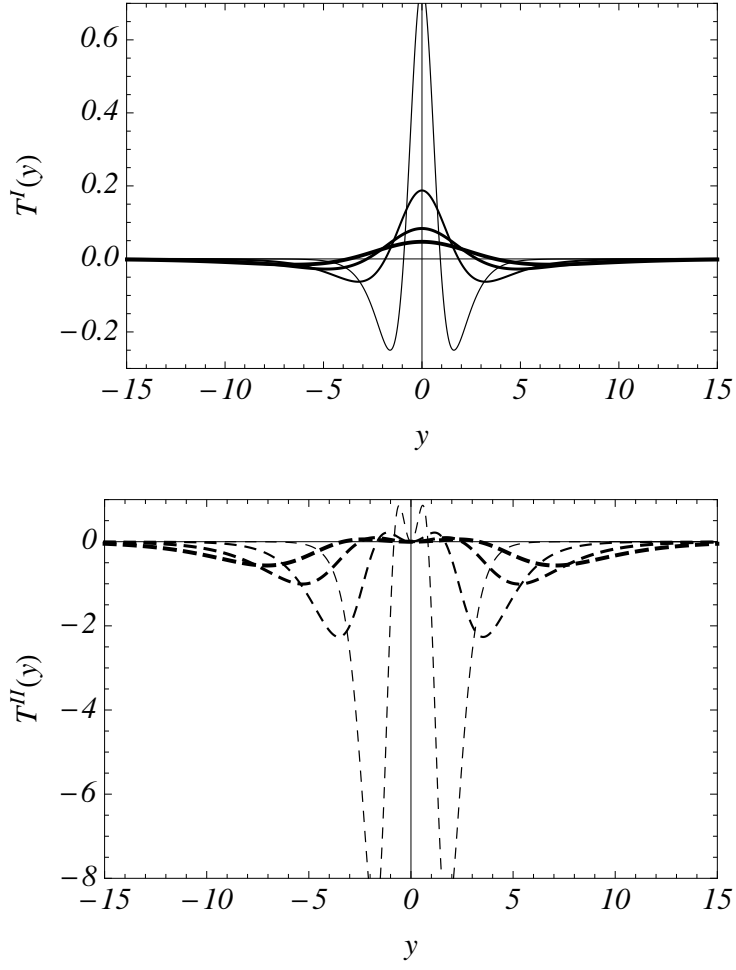
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FIG. 2: Energy density, $T_{00}(y)$, for models I (solid lines) and II (dashed lines) with parameter a running from 1 (thinest line) to 4 (thickest line).

which correspond to the effective real scalar field potentials,

$$V^I(\chi) = \frac{3}{8a^2}\left[1 - 5\sin\left(\sqrt{\frac{2}{3}}\chi\right)^2\right], \tag{39}$$

and

$$V^{II}(\chi) = -\frac{1}{a^2}\left(1 - \frac{\chi^2}{9}\right)\left(9 - \frac{19\chi^2}{8} + \frac{\chi^4}{9}\right). \tag{40}$$

Potentials $V^I(\chi)$ and $V^{II}(\chi)$ suggest the possibility of SSB. However, despite giving rise to the same brane structures, the potentials of the Born-Infeld models, $U^I(\varphi)$ and $U^{II}(\varphi)$, do not hint any SSB. They correspond to a *plateu*-shaped potential with unstable dynamics, with the *plateu*-width proportional to a.

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Once the first-order formalism is adopted, one can map one model into another by setting $A(y) = -\tilde{A}(y)$, instead of constraint Eq. (22), that is, by changing the relative sign between the superpotentials, $w(y)$ and $v(y)$. This would also give rise to AdS domain walls, however, with unlimited energy densities for tachyon fields. As an example, we consider the case of some tachyonic models described through the correspondence with model I (c. f. Eq. (39)), where $A(y) = -\tilde{A}(y)$. This leads to

$$U^I(\psi(y)) = \frac{3}{2a^2} \operatorname{sech}\left(\frac{y}{\sqrt{2}a}\right)^2 \left[\sinh\left(\frac{y}{\sqrt{2}a}\right)^4 + \frac{1}{2}\sinh\left(\frac{y}{\sqrt{2}a}\right)^2\right]^{\frac{1}{2}}, \qquad (41)$$

which corresponds to the solutions of Refs. [29–31], if it is assumed a constraint between the 5-dimensional cosmological constant, Λ_5, and the Hubble expansion rate, H, namely $\Lambda_5 = 6H$.

A second issue to point out concerns the difficulty in establishing the correspondence between tachyon and real scalar field solutions. One could study some deformed topological solutions departing, for instance, from superpotentials like

$$W^{III}(\chi) = \frac{2}{a} \arctan\left[\sinh(\chi)\right], \qquad (42)$$

or

$$W^{IV}(\chi) = \frac{1}{4a}\left[\chi\left(5 - 2\chi^2\right)\sqrt{1-\chi^2} + 3\arctan\left(\frac{\chi}{\sqrt{1-\chi^2}}\right)\right], \qquad (43)$$

which have been considered in discussions about deformed defects [25, 32]. They give rise to the following solutions for $\chi(y)$:

$$\chi^{III}(y) = \operatorname{arcsinh}\left(\frac{y}{a}\right), \qquad (44)$$

$$\chi^{IV}(y) = \frac{y}{\sqrt{a^2+y^2}}, \qquad (45)$$

and, from Eq. (11), the warp factors can be computed,

$$A^{III}(y) = \frac{1}{3}\left[\ln\left(1 + \frac{y^2}{a^2}\right) - 2\frac{y}{a}\arctan\left(\frac{y^2}{a^2}\right)\right], \qquad (46)$$

$$A^{IV}(y) = -\frac{1}{12}\left[\frac{y^2}{\sqrt{a^2+y^2}} + 3\frac{y}{a}\arctan\left(\frac{y}{a}\right)\right], \qquad (47)$$

corresponding to thick brane solutions which induce the stability of the subjacent geometry. However, for cases III and IV the correspondence with tachyonic solutions cannot be established analytically.

Finally, the issue of stability of the above solutions can be verified by assuming that a perturbed metric can be written as

$$ds^2 = e^{2A(y)} \left(\eta_{\mu\nu} + h_{\mu\nu} \right) dx^\mu dx^\nu - dy^2, \tag{48}$$

with $h_{\mu\nu} \equiv h_{\mu\nu}(x, y)$ in the form of transverse and traceless tensor perturbations, for which one has the equation of motion,

$$\left(\frac{d^2}{dy^2} + 4\frac{dA}{dy}\frac{d}{dy} - e^{-2A(y)}\Box \right) h_{\mu\nu}(x, y) = 0, \tag{49}$$

for linearized gravity decoupled to the scalar field, where $\Box \equiv \partial_\mu \partial^\mu$. Assuming a solution

$$h_{\mu\nu}(x, z) = e^{i\,k.x}\, e^{-(3A(z)/2)}\, H_{\mu\nu}(z), \tag{50}$$

with $dz = e^{-A(y)}\, dy$, and dropping the index from $H_{\mu\nu}$, one can transform Eq. (49) into a Schröedinger-like equation,

$$ -H''(z) + V_{(QM)}(z)\, H(z) = k^2\, H(z), \tag{51}$$

such that the localized zero-mode solutions $(k = 0)$ for 4-dimensional gravitational waves can be obtained through the study of the potential

$$V_{(QM)}(z) = \frac{3}{2}A''(z) + \frac{9}{4}A'^2(z), \tag{52}$$

where the primes denote derivative with respect to z. It is possible to state that all the above solutions, from models I to IV, induce stability in the underlying geometry of the problem if $V_{(QM)}$ correspond to volcano-type potentials induced by thick warp factors. Indeed, it can be verified that the zero-modes of models I to IV correspond to the ground-state of $V_{(QM)}$, which gives rise to stable scenarios (i. e. $k^2 > 0$). One thus should expect no tachyonic modes such that no tachyonic condensation would take place.

To summarize, we can say that we have found, through first-order equations of motion, a relationship between a Born-Infeld tachyon solution and the one of a real scalar corresponding to an identical energy density. In what concerns stability, the obtained solutions that we have considered are all stable provided that the effective of volcano-type potential in the associated Schrödinger-like problem leads to normalizable ground state zero-mode wave functions.

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